MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level

List MF27

LIST OF FORMULAE

AND

RESULTS

for Mathematics and Further Mathematics

For use from 2025 in all papers for the H1, H2 and H3 Mathematics and H2 Further Mathematics syllabuses.

This document has **8** pages.

Singapore Examinations and Assessment Board

CAMBRIDGE
International Education

$(2+3x)^5 \nearrow (1+x)^n$

$\searrow (a+b)^n$ ✓

$(1+3x)^5 \nearrow (1+x)^n$

$\searrow (a+b)^n$

$(2+3x)^2 = 2^2 + 2(2)(3x) + (3x)^2$

n=0 → 1
n=1 → 1 1
n=2 → 1 2 1
n=3 → 1 3 3 1
n=4 → 1 4 6 4 1

$(2+3x)^4 = \binom{4}{0}(2)^4(3x)^0 + \binom{4}{1}(2)^3(3x)^1 + \binom{4}{2}(2)^2(3x)^2 + \binom{4}{3}(2)^1(3x)^3 + \binom{4}{4}(2)^0(3x)^4$

$(2+3x)^{-4}$ $\quad (2+3x)^{\frac{1}{2}} = \sqrt{2+3x}$

$n(n-1)(n-2)(n-3)!$

$150! = 150(149)(148)(147)146!$

$$\binom{n}{3} = \frac{n!}{3!(n-3)!}$$

$$= \frac{n(n-1)(n-2)\cancel{(n-3)!}\ 1}{3!\cancel{(n-3)!}\ 1}$$

$$= \frac{n(n-1)(n-2)}{3!}$$

$\sqrt{1+3x}$

ascending.

$= (1+3x)^{\frac{1}{2}}$

$= 1 + \frac{1}{2}(3x) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(3x)^2 + \dots$

$(1+3x)^{\frac{1}{2}}$

$= \left[3x\left(\frac{1}{3x}+1\right)\right]^{\frac{1}{2}}$

$= (3x)^{\frac{1}{2}}\left(1+\frac{1}{3x}\right)^{\frac{1}{2}}$

$= \sqrt{3}\sqrt{x}\left[1 + \frac{1}{2}\left(\frac{1}{3x}\right) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}\left(\frac{1}{3x}\right)^2 + \dots\right]$

$= \sqrt{3}\sqrt{x}\left[1 + \frac{1}{6}x^{-1} - \frac{1}{72}x^{-2} + \dots\right]$

1×10^{-2}

x is small $\quad x = 0.01 \Rightarrow x^2 = 0.0001$, $x^3 = 0.000001$

$\boxed{\text{MF 27}}$ $\boxed{\tan x \simeq x}$

$\tan x = \frac{\sin x}{\cos x}$

Show that $\tan x \simeq x + \frac{1}{3}x^3$ when x is small.

$\simeq \frac{x - \frac{x^3}{3!}}{1 - \frac{x^2}{2!}}$

$\simeq x\left(1-\frac{x^2}{6}\right)\left(1-\frac{1}{2}x^2\right)^{-1}$

$\simeq x\left(1-\frac{x^2}{6}\right)\left(1+(-1)\left(-\frac{1}{2}x^2\right)+\dots\right)$

$\simeq x\left(1-\frac{x^2}{6}\right)\left(1+\frac{1}{2}x^2\right)$

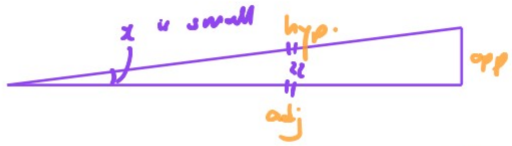
$\simeq x\left(1+\frac{1}{2}x^2-\frac{1}{6}x^2+\dots\right)$

$\simeq x\left(1+\frac{1}{3}x^2\right)$

$\tan x \simeq x + \frac{1}{3}x^3$ #

As x is small,

$\tan x \simeq \sin x$



x is small $\Rightarrow$ hyp $\simeq$ adj.

$\sin x = \frac{\text{opp}}{\text{hyp}} \qquad \tan x = \frac{\text{opp}}{\text{adj}}$

$\boxed{\sin x \simeq \tan x} \simeq x$

Algebraic series

$\binom{n}{1}=n \quad \binom{n}{2}=\frac{n(n-1)}{2!} \quad \boxed{\binom{n}{3}=\frac{n(n-1)(n-2)}{3!}}$

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \binom{n}{3} a^{n-3}b^3 + \ldots + b^n, \text{ where } n \text{ is a positive integer}$$

and $\boxed{\binom{n}{r} = \frac{n!}{r!(n-r)!}}$

$\sin(1+x) = \cancel{a} + \cancel{b}x + \cancel{c}x^2 + \cancel{d}x^3 + \ldots$

$r=3$

Maclaurin expansion:

$\boxed{x \text{ is small} \quad \tan x \approx x}$

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^n}{n!}\mathrm{f}^{(n)}(0) + \ldots$$

$\left|\frac{x}{2}\right| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad \boxed{(|x|<1)}$$

x with $2x$ ✓

x with x^2+3x ✓

x with $3-x$ ⊗

x with $3x^2-1$ ⊗

$$\mathrm{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{(all } x\text{)}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{(all } x\text{)}$$

$\sin(1+x) = (1+x) - \frac{(1+x)^3}{3!} + \frac{(1+x)^5}{5!} - \ldots$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^r x^{2r}}{(2r)!} + \ldots \qquad \text{(all } x\text{)}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^{r+1}x^r}{r} + \ldots \qquad \boxed{(-1 < x \leqslant 1)}$$

$\sin(1+x) = \sin 1 \cos x + \cos 1 \sin x$

Partial fractions decomposition

Non-repeated linear factors:

⊗ $\boxed{\frac{x^2}{(x+1)(3x-1)}}$

proper fraction, degree, constant

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$$

linear

Repeated linear factors:

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C \cancel{+Dx}}{(cx+d)^2}$$

$D=0$

Non-repeated quadratic factor:

liner

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(x^2+c^2)}$$

linear — quadratic

R-formulae

$$3\cos x - 2\sin x = R\cos(x+\alpha)$$

$$= R(\cos x\cos\alpha - \sin x\sin\alpha)$$

$$3\cos x - 2\sin x = (R\cos\alpha)\cos x - (R\sin\alpha)\sin x$$

$R\cos\alpha = 3$ — (1)

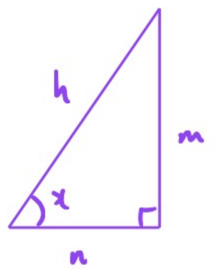
$R\sin\alpha = 2$ — (2)

$(1)^2+(2)^2$: $R^2(\cos^2\alpha+\sin^2\alpha) = 3^2+2^2$

$R = \sqrt{3^2+2^2}$

$\frac{(2)}{(1)}: \frac{R\sin\alpha}{R\cos\alpha} = \frac{2}{3} \Rightarrow \tan\alpha = \frac{2}{3}$

$\alpha = \tan^{-1}\left(\frac{2}{3}\right)$



$\sin x = \frac{m}{h}$, $\cos x = \frac{n}{h}$

By Pythagoras' Theorem,

$h^2 = m^2 + n^2$

$\div h^2$ $\qquad$ $\div h^2$

$1 = \left(\frac{m}{h}\right)^2 + \left(\frac{n}{h}\right)^2$

$1 = \sin^2 x + \cos^2 x$ $\quad \div\cos^2 x$

$\div\sin^2 x$

$\sec^2 x = \tan^2 x + 1$ ✓

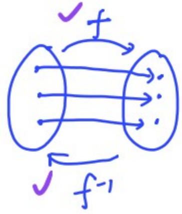
$\operatorname{cosec}^2 x = 1 + \cot^2 x$ ✓

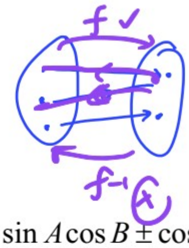
$$\int \tan^2 4x \, dx$$

$\sec^2 A = \tan^2 A + 1$

$$= \int \sec^2 4x - 1 \, dx$$

$$= \frac{\tan 4x}{4} - x + C$$





$\sin^2 A + \cos^2 A = 1$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

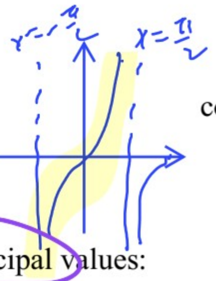
$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2 \sin A \cos A$$ $\Rightarrow \sin \frac{x}{2} = 2 \sin \frac{x}{2} \cos \frac{x}{2}$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$



Principal values:

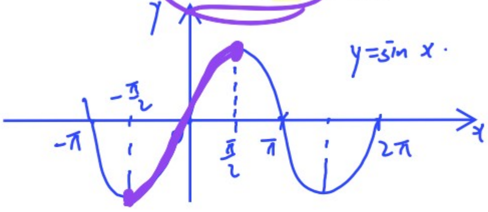
1-1

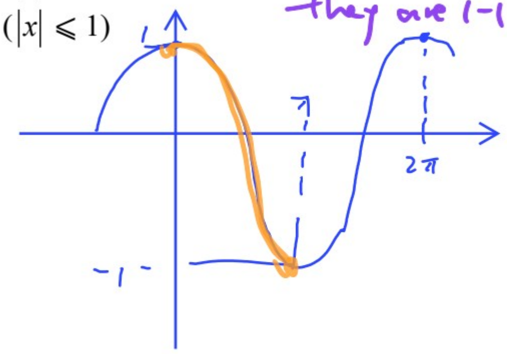
$$-\tfrac{1}{2}\pi \leqslant \sin^{-1} x \leqslant \tfrac{1}{2}\pi \qquad (|x| \leqslant 1)$$

$$0 \leqslant \cos^{-1} x \leqslant \pi \qquad (|x| \leqslant 1)$$

$$-\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

restrict the domain of the trigo curves so that they are 1-1





Derivatives

f(x)		f′(x)
$\sin^{-1} x$	$\rightarrow$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\rightarrow$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\rightarrow$	$\dfrac{1}{1+x^2}$
$\operatorname{cosec} x$	$\leftrightarrows$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\leftrightarrows$	$\sec x \tan x$

Note:

$\int -\operatorname{cosec} x \cot x \, dx = \operatorname{cosec} x + C$

$\int \sec x \tan x \, dx = \sec x + C$

$$\frac{d}{dx}(\operatorname{cosec} x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

$$= \frac{d}{dx}(\sin x)^{-1}$$

$$= -(\sin x)^{-2}(\cos x)$$

$$= -\frac{1}{\sin x}\,\frac{\cos x}{\sin x}$$

$$= -\operatorname{cosec} x \cot x$$

Show

$$\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$$

let $y = \sin^{-1}x$

$\sin y = \frac{x}{1}$

trigo $\rightleftarrows$ Algebra

Diff w.r.t x,

$$\cos y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} \quad ; \quad \cos y = \frac{\sqrt{1-x^2}}{1}$$

$$\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$$

From Derivatives,

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\Rightarrow \int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$$

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right) = \frac{1}{a}\left(\frac{1}{1+\left(\frac{x}{a}\right)^2}\right)$$

Let $x = \frac{u}{a} \Rightarrow \underline{\frac{dx}{du} = \frac{1}{a}} \Rightarrow \underline{dx = \frac{1}{a}\,du}$

$$\int \frac{1}{1+\left(\frac{u}{a}\right)^2}\frac{1}{a}\,du = \tan^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{1+\left(\frac{u}{a}\right)^2}\frac{1}{a}\,du = \tan^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{a^2}{a^2+u^2}\frac{1}{a}\,du = \tan^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx \quad u = \cos x$$

$$= \int \frac{\cancel{\sin x}^{\,1}}{u} \frac{du}{\cancel{\sin x}_{\,-1}} = -\int \frac{1}{u} du$$

$$= -\ln|u| = -\ln|\cos x| + C$$

$$= \ln|(\cos x)^{-1}| + C$$

$$= \ln|\sec x| + C$$

$$|x|^{-1} = \frac{1}{|x|}$$

$$\parallel$$

$$|x^{-1}| = \left|\frac{1}{x}\right| = \frac{1}{|x|}$$

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx$$

$$= -\ln|\cos x| + C$$

$$= \ln|\sec x| + C$$

$\int \frac{1}{\sqrt{x^2-x-1}}\,dx \qquad \int \frac{1}{1-x-x^2}\,dx \qquad \int \frac{1}{\sqrt{3-x^2}}\,dx$

$a^2=3$, $a=\sqrt{3}$, $|x|<\sqrt{3}$

Integrals

(Arbitrary constants are omitted; a denotes a positive constant.)

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x$	
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$	$(\lvert x\rvert < a)$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right)$	$(x>a)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$	$(\lvert x\rvert < a)$
$\tan x$	$\ln(\sec x)$	$(\lvert x\rvert < \frac{1}{2}\pi)$
$\cot x$	$\ln(\sin x)$	$(0<x<\pi)$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x+\cot x)$	$(0<x<\pi)$
$\sec x$	$\ln(\sec x+\tan x)$	$(\lvert x\rvert < \frac{1}{2}\pi)$

partial fraction

cancelling

Q4 & Q1

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

Q1 $\cos x > 0$

Q4 $\cos x > 0$

slow

$A=\frac{1}{2a}$, $B=-\frac{1}{2a}$

$\int \frac{1}{(x-a)(x+a)}\,dx$

$= \int \frac{A}{x-a} + \frac{B}{x+a}\,dx$

$= \frac{1}{2a}\int \frac{1}{x-a} - \frac{1}{x+a}\,dx$

$= \frac{1}{2a}\left[\ln|x-a| - \ln|x+a|\right] + C$

$= \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

Vectors

The point dividing AB in the ratio $\lambda:\mu$ has position vector $\frac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$

Vector product:

$$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3-a_3b_2 \\ a_3b_1-a_1b_3 \\ a_1b_2-a_2b_1 \end{pmatrix}$$

Applications of definite integrals

Arc length of a curve defined in cartesian form: $\int_a^b \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$

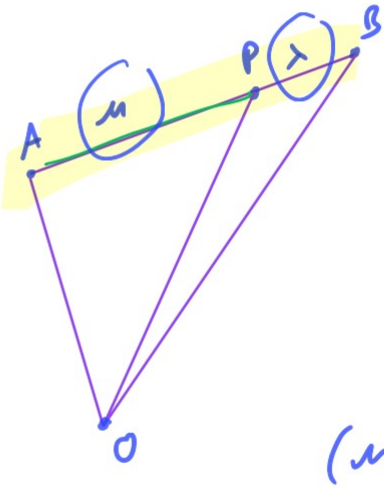
Surface area of revolution about the x-axis for a curve defined in cartesian form:

$$\int_a^b 2\pi y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$$

Functions of two variables

Quadratic approximation of f at (a, b):

$$\mathrm{f}(x,y) \approx \mathrm{f}(a,b) + \mathrm{f}_x(a,b)(x-a) + \mathrm{f}_y(a,b)(y-b) + \frac{1}{2}\mathrm{f}_{xx}(a,b)(x-a)^2 + \mathrm{f}_{xy}(a,b)(x-a)(y-b) + \frac{1}{2}\mathrm{f}_{yy}(a,b)(y-b)^2$$



Note P divides AB in ratio $\mu : \lambda$ ← diff from MF27.

$$\vec{AP} = \frac{\mu}{\mu+\lambda}\vec{AB}$$

$$\vec{OP} - \vec{OA} = \frac{\mu}{\mu+\lambda}\left(\vec{OB} - \vec{OA}\right)$$

$$(\mu+\lambda)\vec{OP} - (\mu+\lambda)\vec{OA} = \mu\vec{OB} - \mu\vec{OA}$$

$$(\mu+\lambda)\vec{OP} = \mu\vec{OB} - \mu\vec{OA} + (\mu+\lambda)\vec{OA}$$

$$\vec{OP} = \frac{\mu\vec{OB} + \lambda\vec{OA}}{\mu+\lambda}$$

Formulae not given in MF27

- $\cos^2 A + \sin^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $\cot^2 A + 1 = \operatorname{cosec}^2 A$

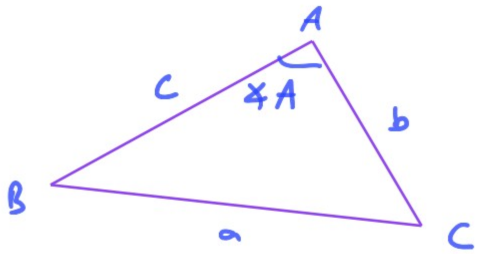
- R-formulae

$s = \frac{\theta^\circ}{360^\circ}(2\pi r)$ $\boxed{\text{or}}$ $s = \frac{\theta}{2\pi}(2\pi r) = r\theta$

$A = \frac{\theta}{2\pi}(\pi r^2) = \frac{\theta r^2}{2}$

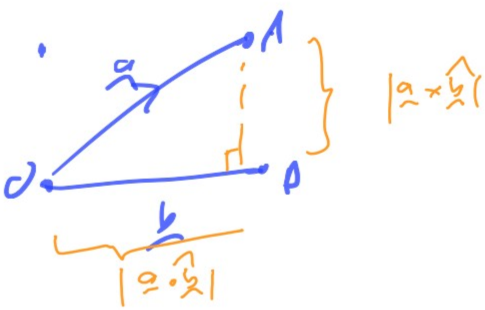
- Arc length, $s = r\theta$
- Area of sector, $A = \frac{1}{2} r^2 \theta$

- cosine rule $\Rightarrow a^2 = b^2 + c^2 - 2bc \sin A$



Generally, more known lengths

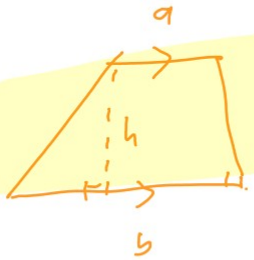
- sine rule $\Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ [more known ∠s]

- All the other vectors formulas
  - $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}|\,|\underset{\sim}{b}| \cos\theta$
  - $\underset{\sim}{a} \times \underset{\sim}{b} = |\underset{\sim}{a}|\,|\underset{\sim}{b}| \sin\theta \; \hat{\underset{\sim}{n}}$
  - 

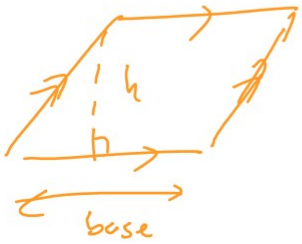
$\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{d}$; $\lambda \in \mathbb{R}$

$\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{d}_1 + \mu \underset{\sim}{d}_2$; $\lambda, \mu \in \mathbb{R}$

$\underset{\sim}{r} \cdot \underset{\sim}{n} = k$

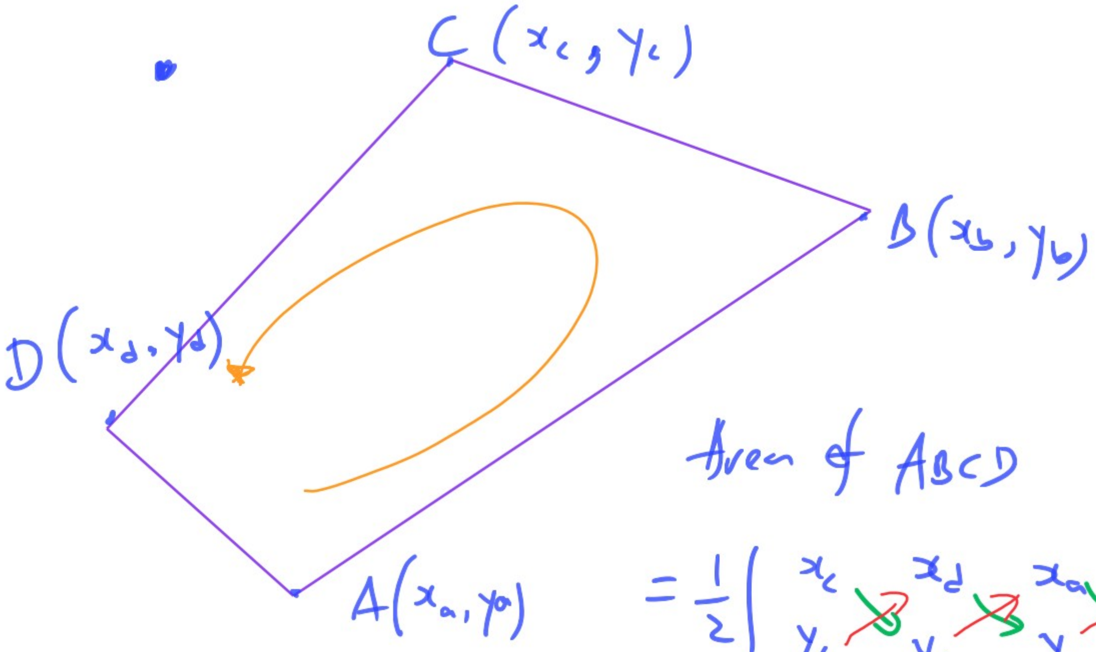
- 

Area $= \left(\frac{a+b}{2}\right)h$



Area $= bh$.

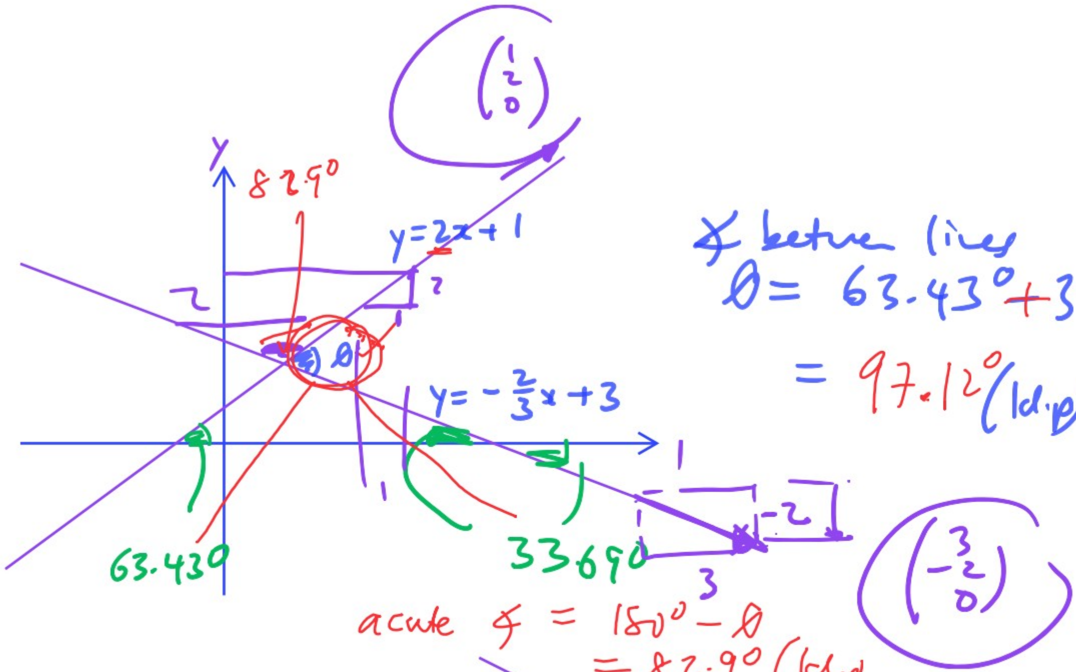
Volume of pyramid $= \frac{1}{3}(BA)(h)$

- 

Area of ABCD

$$= \frac{1}{2}\left| \begin{matrix} x_c & x_d & x_a & x_b & x_c \\ y_c & y_d & y_a & y_b & y_c \end{matrix} \right|$$

$$= \frac{1}{2}\left| (x_c y_d + x_d y_a + x_a y_b + x_b y_c) - (y_c x_d + y_d x_a + y_a x_b + y_b x_c) \right|$$



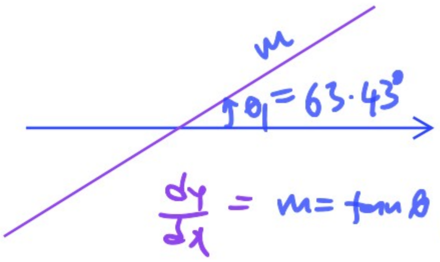
∢ between lines

$\theta = 63.43^0 + 33.69^0$

$= 97.12^0$ (1d.p)

acute ∢ $= 180^0 - \theta$

$= 82.9^0$ (1d.p)

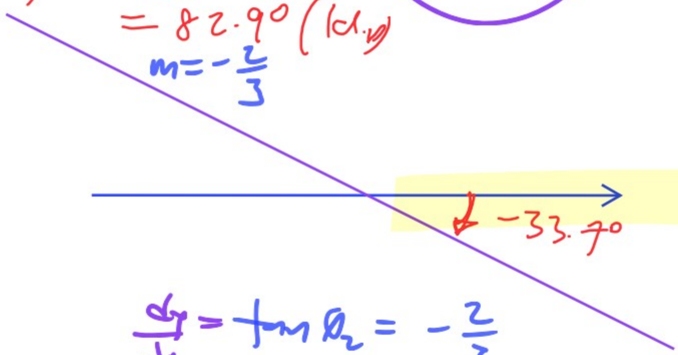


$\frac{dy}{dx} = m = \tan\theta_1$

$m = 2 \Rightarrow \tan\theta_1 = 2$

$\theta_1 = \tan^{-1}(2) = 1.107\text{rad}$

$= 63.43^0$



$\frac{dy}{dx} = \tan\theta_2 = -\frac{2}{3}$

$\theta_2 = \tan^{-1}\left(-\frac{2}{3}\right)$

$= -0.5880\text{rad}$

$= -33.69^0$

Numerical methods

Trapezium rule (for single strip): $\int_a^b \mathrm{f}(x)\mathrm{d}x \approx \frac{1}{2}(b-a)\left[\mathrm{f}(a)+\mathrm{f}(b)\right]$

Simpson's rule (for two strips): $\int_a^b \mathrm{f}(x)\mathrm{d}x \approx \frac{1}{6}(b-a)\left[\mathrm{f}(a)+4\mathrm{f}\left(\frac{a+b}{2}\right)+\mathrm{f}(b)\right]$

The Newton-Raphson iteration for approximating a root of $\mathrm{f}(x) = 0$:

$$x_2 = x_1 - \frac{\mathrm{f}(x_1)}{\mathrm{f}'(x_1)},$$

where x_1 is a first approximation.

Euler Method with step size h:

$$y_2 = y_1 + h\mathrm{f}(x_1, y_1)$$

Improved Euler Method with step size h:

$$u_2 = y_1 + h\mathrm{f}(x_1, y_1)$$
$$y_2 = y_1 + \frac{h}{2}\left[\mathrm{f}(x_1, y_1) + \mathrm{f}(x_2, u_2)\right]$$

Standard discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $\mathrm{Po}(\lambda)$	$\mathrm{e}^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ
Geometric $\mathrm{Geo}(p)$	$(1-p)^{x-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$

Standard continuous distribution

Distribution of X	p.d.f.	Mean	Variance
Exponential	$\lambda \mathrm{e}^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

σ_x , σ , s population std.

sample

$$\boxed{s^2 = \frac{n}{n-1}\sigma_x^2}$$

sample std.

Sampling and testing

Unbiased estimate of population variance:

$$s^2 = \frac{n}{n-1}\left(\frac{\Sigma(x-\bar{x})^2}{n}\right) = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$

Regression and correlation

Estimated product moment correlation coefficient:

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\left\{\Sigma(x-\bar{x})^2\right\}\left\{\Sigma(y-\bar{y})^2\right\}}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}}$$

Estimated regression line of y on x:

$$y - \bar{y} = b(x - \bar{x}), \quad \text{where } b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

WILCOXON SIGNED RANK TEST

P is the sum of the ranks corresponding to the positive differences,
Q is the sum of the ranks corresponding to the negative differences,
T is the smaller of P and Q.

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of T

	Level of significance			
One	0.05	0.025	0.01	0.005
Two	0.1	0.05	0.02	0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

AM-GM inequality:
For any nonnegative real numbers $x_1, x_2, \ldots, x_n$,

$$\frac{x_1 + x_2 + \ldots + x_n}{n} \geqslant \sqrt[n]{x_1 x_2 \ldots x_n}\ ,$$

where the equality holds if and only if $x_1 = x_2 = \ldots = x_n$.

Cauchy-Schwarz inequality:
For any real numbers $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$,

$$\left(\sum_{i=1}^{n} u_i v_i\right)^2 \leqslant \left(\sum_{i=1}^{n} u_i^{\ 2}\right)\left(\sum_{i=1}^{n} v_i^{\ 2}\right),$$

where the equality holds if and only if there exists a nonzero constant k such that $u_i = kv_i$ for all $i = 1, 2, \ldots, n$.

Triangle inequality:
For any real numbers $x_1, x_2, \ldots, x_n$,

$$|\, x_1 + x_2 + \ldots + x_n \,| \leqslant |\, x_1 \,| + |\, x_2 \,| + \ldots + |\, x_n \,|,$$

where the equality holds if $x_1, x_2, \ldots, x_n$ are all nonnegative.

Inclusion-Exclusion Principle:
For any subsets $A_1, A_2, \ldots, A_n$ of a set,

$$\begin{aligned}
|\, A_1 \cup A_2 \cup \ldots \cup A_n \,| = {} & |\, A_1 \,| + |\, A_2 \,| + \ldots + |\, A_n \,| \\
& - [|\, A_1 \cap A_2 \,| + |\, A_1 \cap A_3 \,| + \ldots + |\, A_{n-1} \cap A_n \,|] \\
& + [|\, A_1 \cap A_2 \cap A_3 \,| + |\, A_1 \cap A_2 \cap A_4 \,| + \ldots + |\, A_{n-2} \cap A_{n-1} \cap A_n \,|] \\
& \ \vdots \\
& + (-1)^{n-1} \,|\, A_1 \cap A_2 \ldots \cap A_{n-1} \cap A_n \,|
\end{aligned}$$

This booklet is the property of

SINGAPORE EXAMINATIONS AND ASSESSMENT BOARD