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First published 2024

ISBN 978 981 94 0353 0

Printed in Singapore

PREFACE

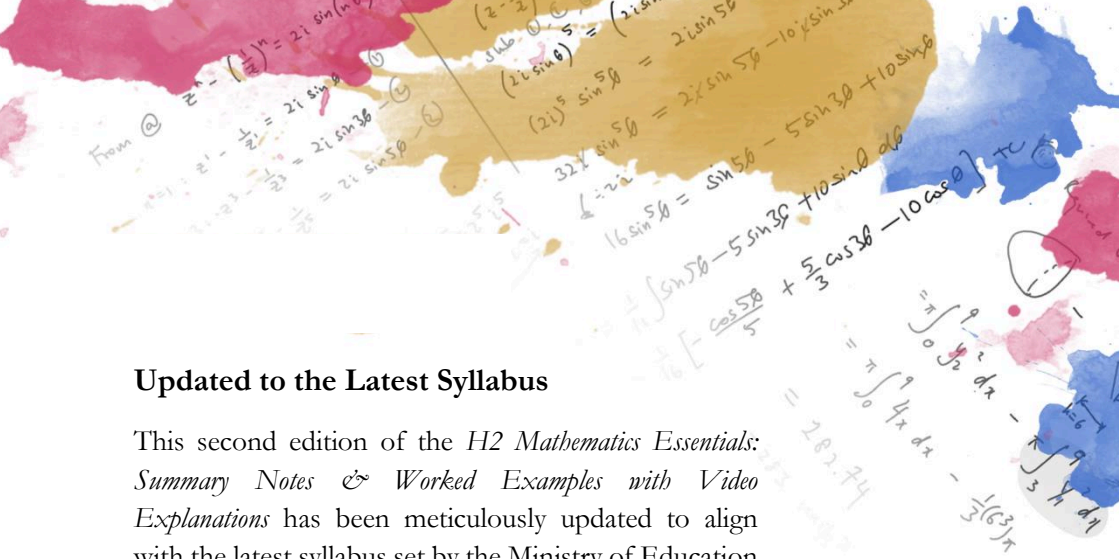
Welcome to the second edition of *H2 Mathematics Essentials: Summary Notes & Worked Examples with Video Explanations* (renamed from *H2 Mathematics Summary Notes*). This comprehensive guidebook is designed to support students taking H2 Mathematics (Syllabus 9758) for their A Level examinations. At Tim Gan Math Learning Centre, we understand the rigorous demands of the H2 Mathematics syllabus, and our goal is to provide students with a resource that not only simplifies complex concepts but also enhances their overall learning experience.

Key Features of This Book

Summary Notes: Each topic in the H2 Mathematics syllabus is succinctly summarized to highlight key concepts, formulas, and theorems. These notes serve as a quick reference to reinforce your understanding and aid in effective revision.

Worked Examples: To bridge the gap between theory and practice, we have included numerous worked examples for each topic. These examples illustrate the application of mathematical principles and problem-solving techniques, providing a clear path from problem statement to solution.

Video Explanations: Understanding that students have diverse learning preferences, we have incorporated QR codes that link directly to video explanations of the worked examples. These videos are taught by our highly experienced and qualified H2 Mathematics tutor, Mr. Timothy Gan. Through these video tutorials, Mr. Gan offers step-by-step guidance, elucidating complex problems and offering insightful tips to tackle various types of questions.



Updated to the Latest Syllabus

This second edition of the *H2 Mathematics Essentials: Summary Notes & Worked Examples with Video Explanations* has been meticulously updated to align with the latest syllabus set by the Ministry of Education (MOE). We have ensured that all topics, examples, and explanations reflect the most current curriculum requirements, providing you with the most relevant and accurate study material.

About Tim Gan Math Learning Centre

At Tim Gan Math Learning Centre, we are committed to fostering a deep understanding and appreciation of mathematics among our students. Our approach combines rigorous academic instruction with personalized support, aimed at helping each student achieve their fullest potential. With years of experience and a proven track record of success, Mr. Timothy Gan and our team of dedicated educators are here to guide you through your H2 Mathematics journey.

Our Commitment

We believe that every student has the capacity to excel in mathematics with the right resources and guidance. *H2 Mathematics Essentials: Summary Notes & Worked Examples with Video Explanations* is crafted with this belief in mind, offering a holistic learning tool that caters to different learning styles and needs. Whether you are seeking to solidify your foundational knowledge, refine your problem-solving skills, or achieve excellence in your examinations, this book is your trusted companion.

We wish you the very best in your studies and hope that this guidebook will be instrumental in your success.

Check Out Our Free Resources for H2 Math

H2 Math Vectors Mini-Course

100 Essential H2 Math Questions

100 H2 Math Integrals

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PURE MATHEMATICS

STATISTICS

PURE MATHEMATICS

"Pure Mathematics is, in its way, the poetry of logical ideas."

- Albert Einstein

INEQUALITIES

BASIC RESULTS

1. If $a > b$ and $b > c \Rightarrow a > c$

   E.g. $12 > 5$ and $5 > -1 \Rightarrow 12 > -1$

2. If $a > b \Rightarrow a \pm c > b \pm c$

   E.g. $-5 > 7 \Rightarrow -5 \pm 10 > 7 \pm 10$

3. If $a > b$ and $c > 0 \Rightarrow ac > bc$; $\dfrac{a}{c} > \dfrac{b}{c}$

   E.g. $21 > 9 \Rightarrow \dfrac{21}{2} > \dfrac{9}{2}$

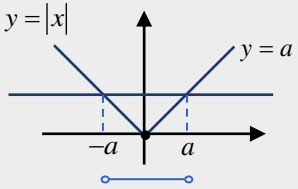
4. If $a > b$ and $c < 0 \Rightarrow ac < bc$; $\dfrac{a}{c} < \dfrac{b}{c}$

   E.g. $9 < 21 \Rightarrow -\dfrac{9}{2} > -\dfrac{21}{2}$

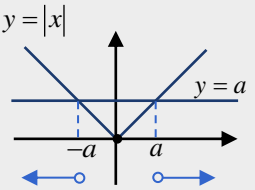
5. If $ab > 0 \Rightarrow$ "$a < 0$ and $b < 0$" or "$a > 0$ and $b > 0$"
6. If $ab < 0 \Rightarrow$ "$a < 0$ and $b > 0$" or "$a > 0$ and $b < 0$"
7. $a < x < b \Leftrightarrow a < x$ **and** $x < b$.
8. Know the difference between **"and"** (intersection of sets) and **"or"** (union of sets).

MODULUS FUNCTION

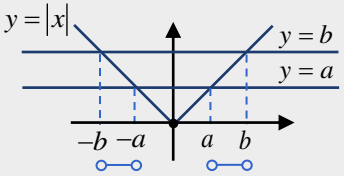
$|x| < a \Leftrightarrow -a < x < a$,
where a is a positive constant.



$|x| > a \Leftrightarrow x < -a$ **or** $x > a$,
where a is a positive constant.



$a < |x| < b \Leftrightarrow -b < x < -a$ **or** $a < x < b$
where a and b are positive constants.



Examples

- $|x| \le 10 \Rightarrow -10 \le x \le 10$
- $|x| > 10 \Rightarrow x > 10$ or $x < -10$
- $|x| < -10 \Rightarrow$ no solution
- $|x| \ge -10 \Rightarrow x \in \mathbb{R}$
- $-5 < |x| < -10 \Rightarrow$ no solution
- $-5 < |x| < 2 \Rightarrow |x| < 2 \Rightarrow -2 < x < 2$
- $2 < |x| < 7 \Rightarrow -7 < x < -2$ or $2 < x < 7$

IMPORTANT NOTES:

1. $x^2 = |x^2| = |x|^2$.
2. If both sides of the inequality are positive, then you can **square** both sides of the inequality. i.e. $|a| < |b| \Leftrightarrow a^2 < b^2$
3. Do not **square** both sides if both sides are **not** positive. i.e. $|x| < x + 1 \not\Rightarrow x^2 < (x+1)^2$. Solve it graphically.

INEQUALITIES INVOLVING SUBSTITUTION

Given that $x < -a$ or $x > b$ where a and b are positive constants.

Substitute x as e^x
$\Rightarrow e^x < -a$ (rej $\because e^x > 0$) or $e^x > b$
$\Rightarrow e^x > b$
$\Rightarrow x > \ln b$

Substitute x as $\ln x$
$\Rightarrow \ln x < -a$ or $\ln x > b$
$\Rightarrow 0 < x < e^{-a}$ or $x > e^b$

Note: $x > 0$ for $\ln x$

Substitute x as $|x|$
$\Rightarrow |x| < -a$ (rej $\because |x| \ge 0$) or $|x| > b$
$\Rightarrow |x| > b$
$\Rightarrow x > b$ or $x < -b$

Given that $-a < x < b$ where a and b are positive constants.

Substitute x as e^x
$\Rightarrow -a < e^x < b$
$\Rightarrow 0 < e^x < b$ since $e^x > 0$
$\Rightarrow x < \ln b$

Substitute x as $\ln x$
$\Rightarrow -a < \ln x < b$
$\Rightarrow e^{-a} < x < e^b$

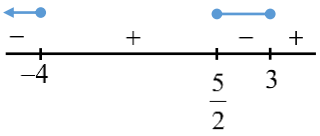
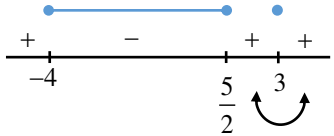
Substitute x as $|x|$
$\Rightarrow -a < |x| < b$
$\Rightarrow 0 \le |x| < b$ since $|x| \ge 0$
$\Rightarrow 0 \le |x|$ and $|x| < b$
$\Rightarrow -b < x < b$

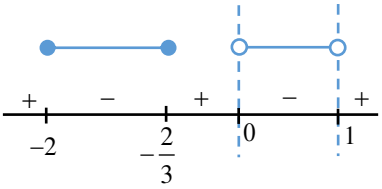
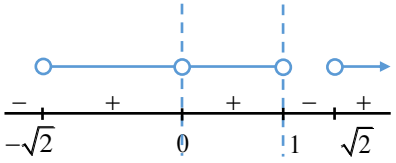
TEST-POINT METHOD GENERAL APPROACH

STEP I	Bring all terms to one side of the equation. (Usually L.H.S) Note: Do not cross-multiply. (Unless it is always positive)
STEP II	Express the terms to a single rational function. Factorise both the numerator and denominator as far as possible. Complete the square for quadratic expression(s) without real roots.
STEP III	Label critical values and roots along the number lines. Test the sign of any interval by substituting a value in the rational function. Alternate the sign for **odd** powered factor. **Do not** alternate the sign for **even** powered factor. (Repeated root)
STEP IV	Read the solution set from the number line. Note: Do not equate to the roots from denominator.

IMPORTANT NOTES:

1. Do not multiply inequality equations with variables without knowing whether it's positive or not.
   Examples of variables which **can** be multiplied both sides with: $|x|$, e^{-x}, $(3x-1)^2$
2. Solutions should not be equal to the roots from the denominator.

Examples	Test-point	Solution
$(2x-5)(x-3)(x+4)\le 0$	 Note: Sign alternate between **odd**-powered factor.	$x\le -4$ or $\frac{5}{2}\le x\le 3$
$(2x-5)(x-3)^2(x+4)\le 0$	 Note: Since $x=3$ is a repeated root, sign **doesn't** alternate around it.	$-4\le x\le \frac{5}{2}$ or $x=3$

$\dfrac{(x+2)(3x+2)}{x(x-1)} \le 0$	 Note: **Do not** equate to the roots from the denominator. I.e. $x=0$ and $x=1$ are vertical asymptotes of the function.	$-2 \le x \le -\frac{2}{3}$ or $0 < x < 1$
$\dfrac{x^2-2}{x^2(x-1)} > 0$		$-\sqrt{2} < x < 0$ or $0 < x < 1$ or $x > \sqrt{2}$

UNIT 1 WORKED EXAMPLE 1

Solve algebraically the inequalities:

(i) $x(2-x)(x-3) > 0$

(ii) $(x+5)(2x+1)(x-4) \le 0$

(iii) $(x+5)(x+3)^2(x-4) \ge 0$

(iv) $(3-2x)(2x-5)^2(x+1) \ge 0$

(v) $(3-x^2)(1-x)^2(x^2+2) > 0$

(vi) $x^2(5-x^2)(x^2+x+5) \le 0$

(vii) $\dfrac{(2x-5)(x-1)}{x(x+2)} \ge 0$

(viii) $\dfrac{(x+2)(3x+2)}{x(x-1)} \le 0$

(ix) $\dfrac{x^2+1}{x^2(x-1)} \ge 0$

(x) $\dfrac{x-5-x^2}{(x^2+1)(x+2)} \le 0$

(xi) $\dfrac{(x+2)(3x+2)}{x(x-1)^2} \le 0$

(xii) $\dfrac{(1-x)(3x-5)}{x(x-1)^3} \ge 0$

Answers: (i) $x < 0$ or $2 < x < 3$ (ii) $x \le -5$ or $-0.5 \le x \le 4$ (iii) $x \le -5$ or $x = -3$ or $x \ge 4$ (iv) $-1 \le x \le 1.5$ or $x = 2.5$ (v) $-\sqrt{3} < x < 1$ or $1 < x < \sqrt{3}$ (vi) $x \le -\sqrt{5}$ or $x = 0$ or $x \ge \sqrt{5}$ (vii) $x < -2$ or $0 < x \le 1$ or $x \ge \dfrac{5}{2}$ (viii) $-2 \le x \le -\dfrac{2}{3}$ or $0 < x < 1$ (ix) $x > 1$ (x) $x > -2$ (xi) $x \le -2$ or $-\dfrac{2}{3} \le x < 0$ (xii) $0 < x < 1$ or $1 < x \le \dfrac{5}{3}$